# CALCULATION OF HEAT TRANSFER TO A CONCAVE SURFACE IN A FREE-MOLECULAR GAS STREAM

L. P. Gashtol'd

UDC 533.601.18

The author describes a method of calculating the relaxation process of multiple reflection of a gas stream by a nonconvex surface, allowing for heat transfer and aerodynamic characteristics.

This paper examines a method of calculating the heat transfer (and the aerodynamic) characteristics in external steady-state flow of a one-component structureless gas over a concave body. The flow regime is considered to be free-molecular, and the velocity distribution in the incident stream is considered Maxwellian, corresponding to temperature $T_\infty$. There is severe nonequilibrium between the incident gas stream and the washed surface: the energy $E_\infty^-$, brought by the incident gas stream per unit time per unit body area is much larger than the energy $E_{S1}^+$, carried away per unit area by reflected gas particles if there were full thermal accommodation, i.e., if the gas particles were reflected with a Maxwellian distribution corresponding to the temperature $T_S$ of the washed body surface. The temperature distribution of the body surface is assumed given.

This type of problem arises when one calculates the thermal conditions of bodies moving at high speed in the upper layers of the atmosphere. Very often the flow conditions are close to the limiting case of hyperthermal flow (the Mach number of the incident stream $M_\infty >> 1$). The temperature of the washed surface of the body results from the balance of heat flux at its surface. In flight conditions near earth orbit a considerable part of the heat is removed by radiation, and usually $T_S/T_\infty \leq 1$ [1]. Thus, in these conditions the washed body can be considered "cold" compared with the incident gas stream, i.e., $E_\infty^- >> E_{S1}^+$.

Theoretical calculations of the heat transfer (and the aerodynamic) characteristics for a body in a free-molecular flow are based on knowing the probability law describing the reflection of gas molecules from the surface back into the gas, i.e., on knowing the interaction function $W(\mathbf{v}, \mathbf{u})$. The form of this function depends on a large number of factors describing the properties of the gas and the surface. The contemporary level of knowledge of the physics of the interaction of gas streams with solid surfaces does not allow an accurate theoretical calculation of the function $W(\mathbf{v}, \mathbf{u})$. In specific calculations one must usually employ more or less plausible hypotheses regarding the form of the function, using very simple theoretical models. One's criteria for this choice are the three basic relations which the model functions W must satisfy: the nonnegative and normalization conditions, and the reciprocity relation [2]. These criteria derive from the physical meaning of the interaction function, and one must take them into account in considering flow over bodies of arbitrary shape.

In this paper we introduced an additional requirement, specific to the problem considered (free-molecular flow over a concave body with strong nonequilibrium in the gas—surface system), applicable to models of the function W, thus selecting a class of models with which one can correctly account for the effect of interference on the heat transfer (and the aerodynamic) characteristics in the problem considered. If a Knudsen gas is enclosed in a container, and one perturbs the equilibrium in the gas—surface system, the equilibrium will be restored by collisions of the gas atoms with the surface. After a certain time (the heterogeneous relaxation time) the thermomechanical equilibrium of the gas with the container wall is restored. The heterogeneous relaxation time is determined by the thermal accommodation coefficient $\alpha$, and also by the degree of initial nonequilibrium of the gas—surface system.

---

N.A. Voznesenskii Leningrad Institute of Finance and Economics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 40, No. 5, pp. 800-806, May, 1981. Original article submitted 26 May, 1980.

0022-0841/81/4005-0484$07.50

The feature specific to the flow over a concave body is multiple reflection of gas atoms from the surface. If the incident gas stream is not in energy equilibrium ($E_\infty^- \neq E_{s1}^+$) with the washed surface and the accommodation coefficient $\alpha < 1$, the multiple reflection process can be regarded as a heterogeneous relaxation of the state of the reflected gas streams towards the state of equilibrium with the surface. The degree of nonequilibrium will reduce at each new reflection. Models of the interaction function W assumed for this case should correspond with the relaxation nature of the multiple reflection process. One can obtain an analytical formulation of the proposed criterion by solving the problem by the method of iterations associated with successive computation of the multiple reflections.

Let us consider flow over a concave cavity. The flow process is described by the chain of distribution functions $f_\beta(\mathbf{r}, \mathbf{v})$ ($\beta = 0, 1, 2, \ldots$, $\mathbf{r}$ is the radius vector to the point in space), where $f_\beta(\mathbf{r}, \mathbf{v})$ is the velocity distribution function (and in space) of the gas atoms that have collided $\beta$ times with the cavity surface. Here $f_0(\mathbf{r}, \mathbf{v}) = f_\infty(\mathbf{r}, \mathbf{v})$ is the velocity distribution function in the incident flow. The total distribution function $f(\mathbf{r}, \mathbf{v})$ is determined by the relation

$$f(\mathbf{r},\ \mathbf{v}) = \sum_{\beta=0,\ 1,\ \ldots} f_\beta(\mathbf{r},\ \mathbf{v}),$$

$$f_{\beta+1}(\mathbf{r},\ \mathbf{u}) = \frac{1}{u_n} \int\limits_{v_n<0} |v_n|\, f_\beta(\mathbf{r}_s,\ \mathbf{v})\, W(\mathbf{r}_s,\ \mathbf{v},\ \mathbf{u})\, d\mathbf{v}, \tag{1}$$

$$\mathbf{u} \in \Omega_s(\mathbf{r}),\ \beta = 0,\ 1,\ 2,\ \ldots,$$

where $\Omega_s(\mathbf{r})$ is the region of velocity space containing the velocity $\mathbf{u}$ with which the atoms travel to the point $\mathbf{r}$ from points $\mathbf{r}_s$ of the cavity surface; and $\mathbf{n}$ is the normal to the surface at the point $\mathbf{r}_s$. (All the quantities used in this paper are dimensionless, and referenced to the appropriate parameters of the incident flow or to the characteristic dimension of the problem.)

If $f_\infty(\mathbf{r}, \mathbf{v})$ and $W(\mathbf{r}_s, \mathbf{v}, \mathbf{u})$ are known, then the whole set of functions $f_\beta(\mathbf{r}, \mathbf{u})$, $\beta = 1, 2, \ldots$ can be determined sequentially with the aid of Eq. (1).

We can now consider two sequences of flows reflected from the cavity surface, of energies $E_\beta^+$, $E_{s\beta}^+$, where

$$E_\beta^+ = \int\limits_S \int\limits_{u_n>0} u_n f_\beta(\mathbf{r}_s,\ \mathbf{u})\, u^2\, d\mathbf{u} dS, \qquad \beta = 1,\ 2,\ \ldots; \tag{2}$$

and S is the area of the cavity surface. We obtain an expression for $E_{s\beta}^+$ from Eq. (2) by replacing $f_\beta(\mathbf{r}_s, \mathbf{u})$ by the function

$$f_{s\beta}(\mathbf{r}_s,\ \mathbf{u}) = \frac{1}{u_n} \left[ \int\limits_{v_n<0} |v_n| f_{\beta-1}(\mathbf{r}_s,\ \mathbf{v})\, d\mathbf{v} \right] \frac{2 u_n h_s^2}{\pi} \exp\left[-h_s u^2\right] \tag{3}$$

($h_s = 1/2RT_s$; R is the gas constant), corresponding to reflection in thermal equilibrium with the surface. It is also convenient to introduce a sequence of streams incident on the cavity surface, of energy

$$E_\beta^- = \int\limits_S \int\limits_{v_n<0} |v_n|\, f_\beta(\mathbf{r}_s,\ \mathbf{v})\, v^2 d\mathbf{v} dS,$$

$$\beta = 0,\ 1,\ 2,\ \ldots$$

The heat flux through the cavity surface per unit time is computed from the formula

$$q = \sum_{\beta=1,2\ldots} (E_{\beta-1}^- - E_\beta^+) = \sum_{\beta=1,2,\ \ldots} E_{\beta-1}^- - \sum_{\beta=1,2\ ,\ldots} E_\beta^+ = E^- - E^-,$$

where $E^-$ is the energy brought to the cavity surface per unit time by the incident gas molecules, and $E^+$ is the energy carried away from the cavity surface by reflected gas molecules.

To describe the completeness of the heat transfer between a gas stream and a surface, one usually employs the thermal accommodation coefficient

$$\alpha = \frac{E^- - E^+}{E^- - E_s^+},$$

where

$$E_s^+ = \sum_{\beta=1,2,\ldots} E_{s\beta}^+.$$

The completeness of the heat transfer in the multiple reflection process can be described by the sequence of accommodation coefficients

$$\alpha_\beta = \frac{E_{\beta-1}^- - E_\beta^+}{E_{\beta-1}^- - E_{s\beta}^+}, \quad \beta = 1,\ 2,\ \ldots,$$

which gives a more detailed evaluation of the adaptation of the reflected gas streams to the surface conditions in the multiple reflection process than does the complete accommodation coefficient $\alpha$.

The connection between $\alpha$ and $\alpha_\beta$ is given by the relation

$$q = E^- - E^+ = \alpha(E^- - E_s^+) = \sum_{\beta=1,2,\ldots} \alpha_\beta (E_{\beta-1}^- - E_{s\beta}^+).$$

However, to solve this problem it is more convenient to use a simpler characteristic of the degree of heat transfer between the gas stream and the surface. For example, one can introduce the sequence of coefficients

$$\lambda_\beta = \frac{E_{s\beta}^+}{E_\beta^+}, \quad \beta = 1,\ 2,\ \ldots \tag{4}$$

If the gas stream reaches thermal equilibrium with the surface after a $\beta$-fold reflection from the surface, then the coefficient $\lambda_\beta$ will be 1. The closer is $\lambda_\beta$ to 1, the more has the gas stream, reflected $\beta$ times from the body surface, adapted towards the surface temperature.

The values of the coefficients of Eq. (4) depend on the form of the interaction function W. The chosen model of this function must ensure the establishment of thermal equilibrium between the gas streams and the cavity surface in the multiple reflection process. This requirement can be expressed analytically in the form of the relation:

$$\lim_{\beta\to\infty} \lambda_\beta = 1. \tag{5}$$

Achievement of the limit of Eq. (5) is a necessary (but not a sufficient) condition for the chosen interaction function model being suitable for solution of the problem. Therefore, requirement (5) can be used as an additional criterion for choice of the models. Models of the function W for which Eq. (5) does not hold cannot be used in cases where multiple reflections of the gas streams make an appreciable contribution to the heat transfer characteristics and the aerodynamic characteristics.

Many papers have been published on computing the heat transfer and aerodynamic characteristics of concave bodies in a free-molecular gas stream. The calculations were done using simplified models of the function for interaction of the gas particles with the surface. However, in these investigations the authors did not check that Eq. (5) holds for the models used, and indeed this requirement was never introduced explicitly. The result is that in many papers models of the function W were used for which Eq. (5) does not hold. Therefore, their results relating to calculating the influence of multiple reflections are unreliable.

It is important to check that Eq. (5) holds for the most widely used models of the interaction function W.

The simplest model of the interaction function can be constructed on the premise of elastic reflection of gas particles from the surface. In this case the function W has the following form [3]:

$$W_e = \frac{\omega(\mathbf{r}_s,\ \mathbf{v}/v,\ \mathbf{u}/u)}{v^2}\,\delta(u - v), \tag{6}$$

where $\delta$ is the Dirac delta function; $\omega(\mathbf{r}_s\ \frac{\mathbf{v}}{v},\ \frac{\mathbf{u}}{u})$ gives the distribution of the reflected particles in direction of travel from the surface, and here

$$\int\limits_{u_n>0} \omega(\mathbf{r}_s,\ \mathbf{v}/v,\ \mathbf{u}/u)\, d(\mathbf{u}/u) = 1,$$

and $d(\mathbf{u}/u)$ is an element of solid angle.

The model $W_e$ ensures that Eq. (5) holds only in the case where a gas stream incident from infinity is in thermal equilibrium with the washed surface ($E^-_\infty = E^+_{s1}$), and condition (5) evidently holds. But if the incident stream is not in equilibrium with the washed surface ($E^-_\infty \neq E^+_{s1}$), then $\alpha_\beta = 0$ for all $\beta = 1, 2, \ldots$, and Eq. (5) does not hold in this case (for $W = W_e$). Therefore, the scheme with purely elastic reflection (a special case of this scheme is mirror reflection) of gas particles from the surface cannot be used to calculate heat transfer between a concave cavity and a free-molecular gas stream with $E^-_\infty \gg E^+_{s1}$.

Also widely used in calculations is the diffuse reflection scheme

$$W_d = (2/\pi)\, u_n h^2 \exp[-hu^2], \tag{7}$$

where $h = 1/2RT_d$; and $T_d$ is the temperature of the diffusely emitted gas molecules. If we assume that the temperature $T_d$ differs from the surface temperature $T_s (h \neq h_s)$, then the coefficients $\lambda_\beta$ will be equal to the constant $h/h_s$.

For $h \neq h_s$, this constant is not 1, so that condition (5) does not hold in this case, and the gas stream does not relax to equilibrium with the surface in the multiple reflection process. The $W_d$ model ensures that condition (5) holds only where $h = h_s$, when $\lambda_\beta = 1$ for all $\beta = 1, 2, \ldots$; i.e., when complete accommodation occurs even at the first reflection of the gas stream from the surface.

However, if the cavity surface is of medium roughness, the surface atoms are heavier compared with the gas atoms, and a gas atom is not captured by the surface, so that for $E^-_\infty \gg E^+_{s1}$ it is more realistic to postulate that several reflections are necessary to establish equilibrium of the gas stream with the surface. Therefore, in these conditions the diffuse scheme of reflection cannot be used. To calculate the heat transfer between the gas stream and a concave surface in this case, one can use the scheme of elastic-diffuse reflection of gas molecules from the surface.

Let $\sigma$ molecules of the gas stream be reflected diffusely from the surface, and $1 - \sigma$ elastically. Then

$$W_{e\text{-}d} = (1 - \sigma) W_e + \sigma W_d,\ 0 < \sigma < 1. \tag{8}$$

The function (8) contains two parameters, $\sigma$ and h, which can depend on the incident velocity and the surface properties. Condition (5) holds only for $h = h_s$. In this case the accommodation coefficient at the first reflection $\alpha_1$ is equal to $\sigma < 1$, and the state of the reflected gas streams approaches the state of equilibrium with the wall in a uniform manner with respect to $\beta$. For $h \neq h_s$ condition (5) does not hold for model (8).

What has been said above shows the narrowness of the class of interaction schemes which satisfy condition (5). However, this class is broadened considerably if we reject the hypothesis that the interaction function W is independent of $\beta$. Let complete accommodation be reached only as a result of several reflections of the gas stream from the washed surface. Then, in solving the problem it will be correct to describe the process of multiple reflection of the gas stream from a concave surface by introducing a sequence of interaction functions $W_\beta (\beta = 1, 2, \ldots)$, and to write Eq. (1) in the form

$$f_\beta(\mathbf{r},\ \mathbf{u}) = \frac{1}{u_n} \int\limits_{v_n<0} |v_n|\, f_{\beta-1}(\mathbf{r}_s,\ \mathbf{v})\, W_\beta(\mathbf{r}_s,\ \mathbf{v},\ \mathbf{u})\, d\mathbf{v}, \tag{9}$$
$$\mathbf{u} \in \Omega_s(\mathbf{r}),\ \beta = 1,\ 2,\ \ldots$$

For $\beta \to \infty$ the sequence of functions $W_\beta$ must tend to $W_s = (2/\pi)u_n h_s^2 \exp[-h_s u^2]$, corresponding to reflection in thermal equilibrium with the surface, which ensures that condition (5) holds. For example, if we take an arbitrary sequence of numbers $h_\beta$ satisfying the condition $h_\beta \to h_s$ as $\beta \to \infty$, and introduce the sequence of interaction functions

$$W_\beta = (2/\pi)u_n h_\beta^2 \exp[-h_\beta u^2], \tag{10}$$

then condition (5) will clearly hold. Here there is a considerable freedom in choosing the specific values of the parameters $h_\beta$.

By using a sequence of functions $W_\beta$, we can more accurately describe the relaxation of the reflected gas streams towards a state of thermal equilibrium with the washed surface of a concave body in the multiple reflection process.

In view of the above, it is interesting to examine the problem of hypersonic free-molecular gas flow over a cold ($T_s/T_\infty \leq 1$) concave body. In choosing a model for the interaction function W, we should take account of the severe thermal nonequilibrium between the incident gas flow and the washed surface ($E_\infty \gg E^+_{s1}$). We postulate the several reflections of the gas stream from the surface are necessary to establish thermal equilibrium.

For example, we can assume that the gas atoms are reflected from the surface in accordance with an elastic-diffuse reflection scheme with $h = h_s$. Since we can neglect the thermal velocities in this problem (compared with the incident flow velocity), we can drop the second term in Eq. (8), and use the simplified model in the calculations

$$W^* = \sigma_1 W_y, \tag{11}$$

where $\sigma_1 = 1 - \sigma$.

We can also use an interaction function model [3]

$$W^{**} = \frac{\omega(\mathbf{r}_s, \mathbf{v}/v, \mathbf{u}/u)}{(\gamma v)^2} \underset{0<\gamma<1}{\delta(u - \gamma v)}, \tag{12}$$

in which the velocity of an incident gas atom decreases according to the law $u/v = \gamma$ after colliding with the washed surface.

Models (12) and (11) ensure that the state of the reflected gas streams approaches thermal equilibrium with the surface uniformly with respect to $\beta$. Here the relaxation time is determined by the chosen value of the parameters $\sigma_1$ and $\gamma$. In problems where it is invalid to postulate a uniform relaxation process at repeated reflections of the gas stream from the surface one must use more flexible models of the interaction function. In that case it is particularly appropriate to use a sequence of $W_\beta$ and the chain of equations in Eq. (9).

In calculating the characteristics of heat transfer between a light flux and a concave solid surface, one must also compute the relaxation nature of the multiple reflection process. If the light flux is considered as a flux of photons, then the process of multiple reflection of light by the surface can be investigated with the aid of concepts and equations analogous to those used in the aerodynamics of free-molecular flow [4]. In particular, in this approach we use an interaction function for the photons and the surface. In the case of a concave scattering surface the criterion for choosing models of the function can be formulated in the form of Eq. (5).

## NOTATION

W, interaction function for the gas atoms and the body surface; **v**, incident velocity; **u**, velocity of reflection; E, energy; $\alpha$, accommodation coefficient; q, heat flux; T, temperature. Subscripts: $-$ and $\infty$, incident stream; $+$, reflection; s, flow reflected in equilibrium with the surface; $\beta$, multiplicity of reflection.

## LITERATURE CITED

1. M. N. Kogan, Rarefied Gas Dynamics, Plenum Publ. (1969).
2. R. G. Barantsev, Interaction of Rarefied Gases with Surfaces [in Russian], Nauka, Moscow (1975).
3. R. G. Barantsev, L. A. Vasil'ev, and L. P. Gashtol'd, "Optical modeling of the characteristics of a free-molecular gas stream, integrated over momentum," in: Physical Mechanics [in Russian], No. 1, Leningrad State Univ. (1974).

4. E. V. Alekseeva, L. A. Vasil'ev, and L. P. Gashtol'd, "Calculation of the forces and moments due to light pressure on a body," in: Aerodynamics of Rarefied Gases [in Russian], No. 9, Leningrad State Univ. (1978).

---

# DIFFUSION PROBLEM OF A CREEPING GAS.

## I. CALCULATION OF DISTRIBUTION FUNCTIONS OF A MOLECULAR GAS IN THE KNUDSEN LAYER

S. P. Bakanov and V. I. Roldugin

UDC 533.6.011

A modified method of half range expansions has been used to find distribution functions of a molecular gas mixture inhomogeneous in concentration in the Knudsen layer.

The study of dilute gas flow at finite Knudsen numbers can be carried out successfully within hydrodynamics; i.e., within models of a continuous medium, taking into account; however, corrections related to the nonvanishing of the Knudsen number primarily in the boundary conditions. This approach assumes the introduction of "jumps" of macroscopic parameters, such as temperature. gas velocity, etc., at the phase boundary, and the problem reduces to more or less rigorous calculations or estimates of the corresponding coefficients.

Unfortunately, a more general and informative method is that of finding the velocity distribution function of gas molecules, including the layer immediately adjacent to the boundary — the Knudsen layer. This approach requires the solution of a quite complicated kinetic equation, which cannot be done in the general case.

The approximate methods adopted for this purpose consist of either replacing the Boltzmann equation by a model equation with its subsequent solution, or searching an approximate solution of the Boltzmann equation itself by one of the well-known methods. In our opinion, the most fruitful method is that of the half-range expansions, suggested in [1]. Its application in the original form is rendered difficult by the complexity of calculating the half-range moments of the Boltzmann collision integral, integral parentheses (see, e.g., [2]). Papers were later published [3-7], however, in which a modification of the method of [1] was suggested, making it possible to shorten the number of integral brackets requiring direct evaluation, and to simplify the system of moment equations for determining the expansion coefficients of the unknown molecular distribution functions.

This modified method of half-range expansions was used to find molecular distribution functions of a simple gas in the Knudsen layer, which in turn made it possible to accurately calculate the isothermal [6] and thermal [7] coefficients of a creeping gas.

An attempt of applying the half-range expansion method to the calculation of distribution functions for a gas mixture was undertaken in [8, 9]. The author of these papers, however, did not correctly calculate the half-range moments of the Boltzmann integral, a task involving great computational difficulty. In fact, the calculations of these expressions for a gas mixture is incomparably harder than in the case of a simple gas. Therefore, a modification similar to [3-7] is of considerable interest.

A detailed analysis of the procedure of constructing the moment equations for the kinetic Boltzmann equation was performed in [10]. It was shown that the Onsager mutuality principle makes it possible to remove the uncertainty in the choice of moments. We take this fact into account in obtaining a system of equations for determining expansion coefficients in polynomials of velocity distribution functions of components of a binary gas mixture.

In the present paper we establish a number of important relations between the half-range moments of the Boltzmann collision integral for a gas mixture, and we calculate the necessary moments. Based on that, we calculate molecular distribution functions in the Knudsen layer and

---

Institute of Physical Chemistry, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 40, No. 5, pp. 807-817, May, 1981. Original article submitted June 3, 1980.

0022-0841/81/4005-0489$07.50